LECTURE: 3-5 IMPLICIT DIFFERENTIATION (PART 2)

Example 1: Review. Find $\frac{dy}{dx}$ by implicit differentiation.

(a) $x^2 - xy - y^2 = 1$

(b) $\sin(x + y) = 2x - 2y$

Example 2 Find all points on the curve $x^2 + 2y^2 = 1$ where the tangent line has slope 1.

Example 3: If $g(x) + x \sin g(x) = 3x^2 + 1$ and $g(1) = 0$ find $g'(1)$.

Derivatives of Inverse Trigonometric Functions

Implicit differentiation is also used to derive formulas for derivatives of inverse functions.

Example 4: Find the derivatives of the following functions.

(a) $y = \sin^{-1} x$

(b) $y = \tan^{-1} x$

Example 5: Using implicit differentiation find the derivative of $y = \cos^{-1} x$.

Derivatives of Inverse Trigonometric Functions:

- $\dfrac{d}{dx}(\sin^{-1} x) =$ _____
- $\dfrac{d}{dx}(\cos^{-1} x) =$ _____
- $\dfrac{d}{dx}(\tan^{-1} x) =$ _____

Example 6: Differentiate the following functions.

(a) $y = \cos^{-1}(3x + 5)$

(b) $y = \arctan 2x$

Example 7: Differentiate the following functions.

(a) $f(x) = \arcsin(\sqrt{x})$

(b) $g(x) = \tan^{-1}(x - \sqrt{1 + x^2})$

Example 8: Differentiate the following functions.

(a) $y = x^2 \tan^{-1} \sqrt{x}$

(b) $y = x \sin^{-1} x + \sqrt{1 - x^2}$

Example 9: The *van der Waals* equation for n moles of a gas is

$$\left(P + \frac{n^2 a}{V^2}\right)(V - nb) = nRT$$

where P is the pressure, V is the volume, and T is the temperature of the gas. The constant R is the universal gas constant and a and b are constants that are characteristic of a particular gas.

(a) If T remains constant, use implicit differentiation to find dV/dP.

(b) Find the rate of change of volume with respect to pressure of 1 mole of carbon dioxide at a volume of $V = 10$ L and a pressure of $P = 2.5$ atm. Use $a = 3.592$ L^2-atm/mole^2 and $b = 0.04267$ L/mole.